Analysis of Pulsating Pressure Fields in Jets with Respect to Noise Generation

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Nomenclature

c = velocity of sound in atmosphere
m_a = mass flow rate of air in jet inducted from ambient air
m_c' = mass flow rate of air in jet boundary layer contributed by nozzle discharge
m_{co} = mass flow rate of air issuing from nozzle exit
M_c = mass of air in typical cell
q_j = $\frac{1}{2}\rho_j U_j^2$
R_g = gas constant
$\bar{T}$ = mean static temperature of air in jet mixing zone at any given cross section
ρ = density in jet at any given point

Subscripts
1 or j = in jet at nozzle exit
2 = in ambient air

Theme

WITH the objective of contributing to the simplification of methods of analyzing and investigating the effects of jet nozzle geometry and flow conditions on noise, this paper presents a method of computing the pulsating pressure in jets discharging into the atmosphere and indicates how these values may be utilized to compute far-field noise. The method is illustrated for four cases involving different nozzle geometry and flow conditions.

Contents

The modeling of the noise sources in a jet as a field of pulsating pressures proposed by Meecham[1] and Ribner[2] appeared interesting to explore because pulsating pressure is much easier to measure and to manipulate analytically than the Lighthill Tensor. Scharton and White[3] indicated a high correlation between the pulsating pressures measured in a jet and the measured far-field noise and developed an equation for computing the far-field noise from the pulsating pressures in the jet.

In the present paper, a simple model was chosen for the mechanism of pulsating pressure generation and the analysis was limited to subsonic jets, a diagram of which is shown in Fig. 1. Evidence[4] in the literature indicates that as the axially directed kinetic power of the jet is degraded into turbulence by mixing with the ambient air it generates large scale eddies of the order of the thickness of the mixing zone. The transient hydrodynamic processes associated with the birth of these large eddies is obscure. In the present model, it will be assumed that pressure fluctuations occur in the transient process leading to the formation of these large eddies and that they have an amplitude commensurate with the energy being transformed from the axially directed kinetic power in the jet. This kinetic power, designated KP, is given by (see Fig. 1 and Nomenclature)

$$KP = \int_o^{y_2} \pi y \rho U^3 \, dy - \tfrac{1}{2} m_a U_2^2 \tag{1}$$

the value of which at the nozzle exit plane is designated KP_o.

The jet is assumed to be divided longitudinally into a succession of cells each having a length in the axial direction equal to the thickness b of the mixing zone. (Two such cells are illustrated in Fig. 1.) It is further assumed that the energy deposited in a given cell n in the time t_c required to fill the cell with a fresh charge of fluid consists of the energy transformed from kinetic power KP into turbulence in the time t_c within cell n plus α times the energy associated with the pulsating pressure that the fluid contained in the n-1th cell. The energy deposited in a given cell in time t_c by degradation of the

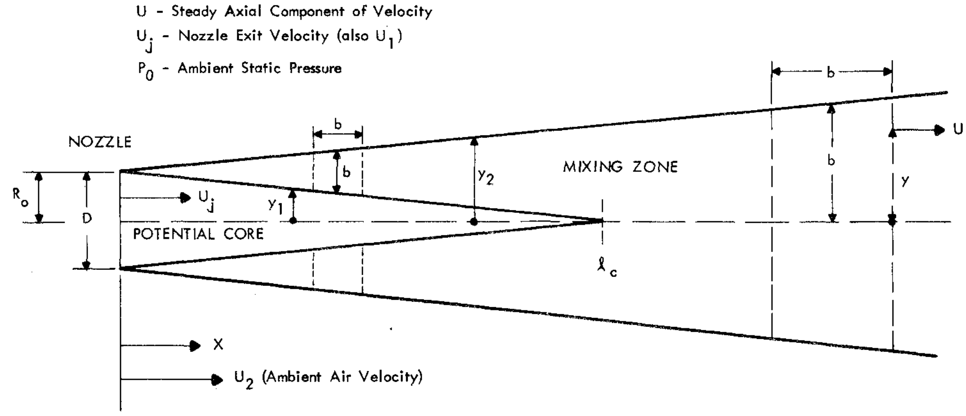

Fig. 1 Illustrative jet.

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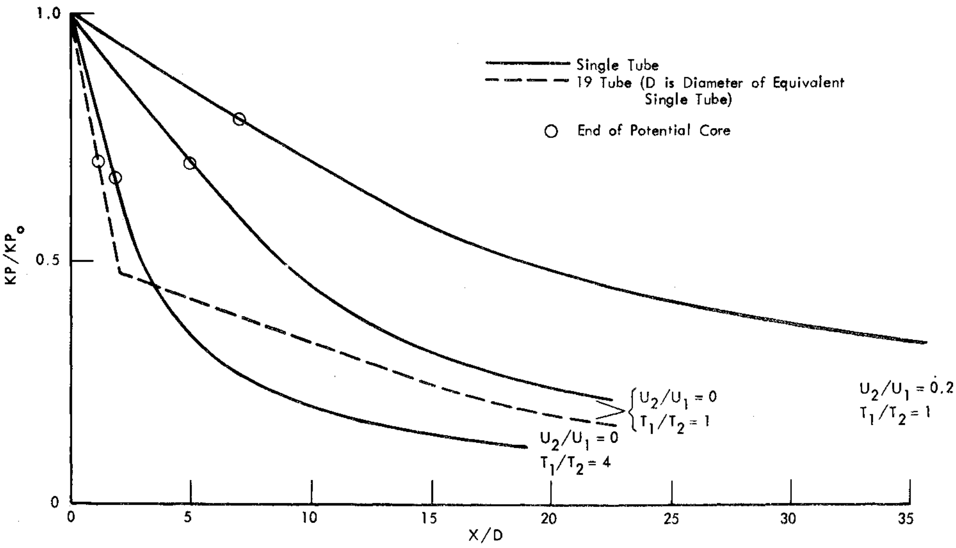

Fig. 2 Effect on axial kinetic power of forward flight speed, jet temperature, and number of tubes in nozzle.

kinetic power directly in that cell is $E_c = -t_c b\, d(KP)/dX$ where t_c may be approximated by $M_c/(m_a+m_c')$. The amplitude of the fluctuating pressure corresponding to the energy E_c is $P_p/P_o = E_c/M_c R_g T$. Define S by the equation $S \equiv (b/D)(m_{co}/\int_{y_1}^{y_2} 2\pi y \rho U dy)$ where the integral is equal to $m_a + m_c'$.

The quantity P_p corresponds to the assumption of α equal to zero. If we define P^* as the value at the pulsating pressure amplitude corresponding to some other value of α and $\psi \equiv P_p^*/P_p$ then from the foregoing equations we obtain

$$P_p^*/q_j = -\psi S(T_j/\overline{T})\, d(KP/KP_o)/d(X/D) \quad (2)$$

It can be shown that the value of ψ can be approximated by

$$\psi \simeq \left[1-\alpha+\alpha \frac{b}{R_o}\frac{d}{dX}\log\left(-\frac{b}{R_o}\frac{d(KP/KP_o)}{d(X/l_c)}\right)\right]^{-1} \quad (3)$$

The velocity profiles U use in computing the illustrative cases were obtained from Ref. 5. Figures 2 and 3 show the values of KP/KP_o and P^*/q_j computed from Eqs. (1) and (2), respectively for four cases exploring the effect of variation in T_1, U_2 and the number of tubes in the nozzle. A value of $\alpha = 0.35$ used in computing P^*/q_j gave good agreement with the limited amount of available experimental data.

For the 19-tube nozzle, the nineteen jets behave essentially as separate jets up to roughly the point in Fig. 2 where the curve breaks and, up to this point, the slope of the curve would be the same as for the single tube nozzle if the diameter of the individual tubes were used in computing X/D. Beyond the break in the curve, the jets gradually coalesce into a single jet.

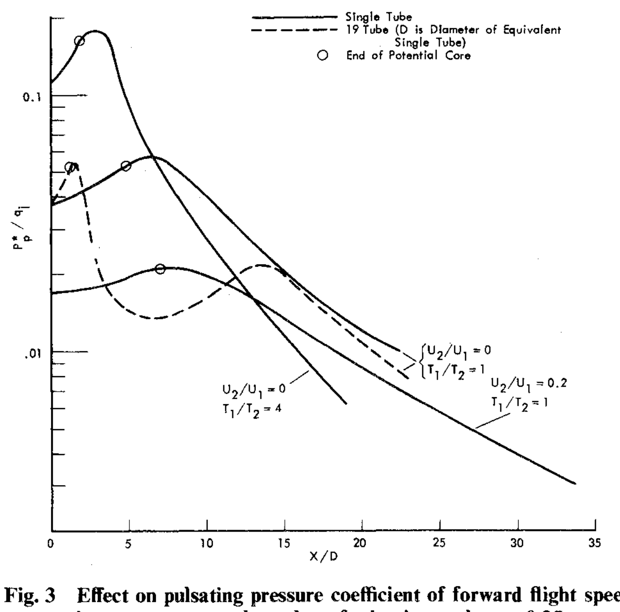

Fig. 3 Effect on pulsating pressure coefficient of forward flight speed, jet temperature and number of tubes in nozzle $\alpha = 0.35$.

An approximate indication of the effect of the pulsating pressures shown in Fig. 3 on overall radiated sound P_s (integrated over frequency) may be obtained from Eq. (11) of Ref. 3 by introducing some simplifying assumptions.

$$P_s \sim \frac{\rho_j^2 U_j^8 R_o^2}{\rho_2 c^5}\left(\frac{l_c}{R_o}\right)\left(\frac{P_p^*}{q_j}\right)_\theta^2 \quad (4)$$

where the subscript θ denotes the maximum value.

Comparison of the values of P_s from Eq. (4) for the case $T_1/T_2 = 4$, $U_2/U_1 = 0$ with that for the case $T_1/T_2 = 1$, $U_2/U_1 = 0$ indicates that P_s varies as the first power of the jet density. This result is in agreement with the prediction by Lighthill.[6] Experimental data in the literature on this point are inconclusive.

In the 19-tube nozzle studied most of the noise generation occurs before the jets coalesce. In this case, Eq. (4) indicates the overall noise power would be nearly the same as for a single nozzle of the same total area (the frequency would of course be higher). Equation (4) does not include the effects of reflection, refraction, and absorption of the noise in passing through the jets which become of increasing importance as jet speeds increase. Equation (4) holds only for the stationary nozzle and was not applied to the case $U_2/U_1 = 0.2$. The far-field sound power can be related directly to the degradation of jet kinetic power by means of Eqs. (2) and (4)

$$P_s \sim \frac{\rho_j^2 U_j^2 R_o^2}{\rho_2 c^5}\left(\frac{l_c}{R_o}\right)\psi^2 S^2 \left(\frac{T_j}{\overline{T}}\right)^2 \left(\frac{d(KP/KP_o)}{d(X/D)}\right)^2 \quad (5)$$

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